The Bearing Capacity of Soils

Dr Omar Al Hattamleh
Example of Bearing Capacity Failure

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The Bearing Capacity of Soils

- Terzaghi’s Ultimate Bearing Capacity
- Meyerhof’s Method
- Brinch Hansen’s Method
- Vesic’s Method
- General Ultimate Bearing Capacity
Classification of foundations

- Foundations
  - Shallow (Chap. 5–10)
    - Spread Footings
    - Mats
  - Deep (Chap. 11–17)
    - Piles
    - Anchors
    - Drilled Shafts
    - Other Types
      - Cassions
      - Pressure-Injected Footings
Spread footing  Shapes & Dimensions

- Square
- Rectangular
- Circular
- Bearing Wall
- Continuous
- Combined
- Ring
Requirement for Foundation

A shallow foundation must:
1. be safe against an overall shear failure in the soil that supports it.
2. cannot experience excessive displacement (in other words, settlement).
3. cannot experience Excessive Lateral Movement.

The definitions of bearing capacity are,

$q_o$ is the contact pressure of the soil at the footing’s invert;
$q_u$ is the load per unit area of the foundation at which the shear failure in soil occurs and is called the ultimate bearing capacity of the foundation; and
$q_{all}$ is the load per unit area of the foundation that is supported without an unsafe movement of the soil, and is called the allowable bearing capacity.
Mode of Failure

A continuous footing resting on the surface of a *dense sand* or a *stiff cohesive* soil is shown in Figure 2a with a width of B. If a load is gradually applied to the footing, its settlement will increase. When the load per unit area equals $q_{ult}$ a sudden failure in the soil supporting the foundation will take place, with the failure surface in the soil extending to the ground surface. This type of *sudden* failure is called a *general shear failure*.

If the foundation rests on sand or clayey soil of *medium compaction* (Figure 2b), an increase of load on the foundation will increase the settlement and the failure surface will *gradually* extend outward from the foundation (as shown by the solid line). When the load per unit area on the foundation equals $q_{ult}$, the foundation movement will be like sudden jerks. A considerable movement of the foundation is required for the failure surface in soil to extend to the ground surface (as shown by the broken lines). The load per unit area at which this happens is the *ultimate bearing capacity* $q_{ult}$. Beyond this point, an increase of the load will be accompanied by a large increase of footing’s settlement. The load per unit area of the footing $q_{ult}$, is referred to as the *first failure load* (Vesic 1963). Note that the peak value of $q$ is not realized in this type of failure, which is called the *local shear failure* in soil.

If the foundation is supported by a fairly *loose* soil, the load-settlement plot will be like the one in Figure 2c. In this case, the failure surface in soil will not extend to the ground surface. Past the value $q_{ult}$, the load-to-settlement plot will be steep and practically linear. This type of failure is called the *punching shear failure*. 
Modes of bearing capacity failure:

(a) Failure surface in soil

Dense sands and stiff cohesive soils.

(b) Failure surface

Soils of medium density or stiffness.

(c) Failure surface

Loose and soft soils.
Modes of failure

Based on experimental results from Vesic (1963), a relation for the mode of bearing capacity failure of foundations can be proposed (Figure 4), where

- $D_r$ is the relative density in sand,
- $D_f$ is the depth of the footing measured from the ground surface,
- $B$ is the width and $L$ is the length of the footing (Note: $L$ is always greater than $B$)

$$B^* = \frac{2BL}{B + L}$$
Range of settlement of circular and rectangular plates at ultimate loads for $Df/B = 0$ in sand (after Vesic, 1973).
Terzaghi’s Bearing Capacity Formulas
Terzaghi’s Ultimate Bearing Capacity Theory

Using an equilibrium analysis, Karl Terzaghi expressed in 1943 the ultimate bearing capacity \( q_u \) of a particular soil to be of the form,

\[
q_u = c N_c + \bar{q} N_q + 0.5 \gamma B N_\gamma
\]

(for strip footings, such as wall foundations)

\[
q_u = 1.3 c N_c + \bar{q} N_q + 0.4 \gamma B N_\gamma
\]

(for square footings, typical of interior columns)

\[
q_u = c N_c + \bar{q} N_q + 0.3 \gamma B N_\gamma
\]

(for circular footings, such as towers, chimneys)

Where,

\( q = \gamma D_f \) is the removed pressure from the soil to place the footing

\( N_c, N_\gamma, \) and \( N_q \) are the soil-bearing capacity factors, dimensionless terms, whose values relate to the angle of internal friction. These values can be calculated when is known or they can be looked up in Terzaghi’s Bearing Capacity Factor Table 3.1 page 87.

\( c' = \) cohesion of soil

\( \gamma = \) unit weight of soil
Terzaghi's Ultimate Bearing Capacity Factors

The bearing capacity factors $N_c$, $N_q$, and $N_\gamma$ are defined by (Table 4.1):

$$N_c = \cot \phi \left[ \frac{e^{2(3\pi/4 - \phi'/2)}\tan \phi'}{2 \cos^2\left(\frac{\pi}{4} + \frac{\phi'}{2}\right)} - 1 \right] = \cot \phi' (N_q - 1)$$

$$N_q = \frac{e^{2(3\pi/4 - \phi'/2)}\tan \phi'}{2 \cos^2\left(45 + \frac{\phi'}{2}\right)}$$

$$N_\gamma = \frac{1}{2} \left( \frac{K_{p\gamma}}{\cos^2 \phi'} - 1 \right) \tan \phi'$$

$$K_{p\gamma} = \tan^2(45 + \phi'/2)$$

### Table 3.1 Terzaghi's Bearing Capacity Factors—Eqs. (3.4), (3.5), and (3.6)

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*From Kumbhojkar (1993)
B.C. Factor of Safety

The factor of safety $FS$ against a bearing capacity failure defined as

$$q_{all} = \frac{q_{ult}}{FS}$$

where $q_{all}$ is the gross allowable load-bearing capacity and $q_{net}$ is the net ultimate bearing capacity.

The factor of safety is chosen according the function of the structure, but never less than $3$ in all cases.

The net ultimate bearing capacity is defined as the ultimate pressure per unit area of the footing that can be supported by the soil in excess of the pressure caused by the surrounding soil at the foundation level.

$$q_{net} = q_{ult} - \bar{q} = q_{ult} - \gamma D_f$$

A footing will obviously not settle at all if the footing is placed at a depth where the weight of the soil removed is equal to the weight of the column’s load plus the footing’s weight.
Modification of the Bearing Capacity Equations for the Water Table

Case I: When $0 < D_1 < D_f$.

$$q_e = D_1 \gamma + D_2 (\gamma_{sat} - \gamma_w)$$

In term 2 of BC equation

Use $\gamma'$ in term 3 of BC equation
Modification of the Bearing Capacity Equations for the Water Table

Case II: When \(0 \leq d \leq B\)

\[
\bar{\gamma} = \gamma' + (\gamma - \gamma') \frac{d}{B}
\]

In term 3 of BC equation

Use \(\gamma\) in term 2 of BC equation
Modification of the Bearing Capacity Equations for the Water Table

Case III. When \( d \geq B \), the water table will have no effect on the ultimate bearing capacity.
The Bearing Capacity for Local or Punching Shear failure

For the local shear failure Terzaghi proposed reducing the cohesion and internal friction angle as

\[ c'' = 0.67c \]
\[ \phi'' = \tan^{-1}(0.67 \tan \phi) \]
Examples (1)

A square foundation is 1.5m x 1.5m in plan. The soil supporting the foundation has a friction angle of $\phi = 20^\circ$ and $c' = 65\text{kPa}'$. The unit weight of soil is 19kN/m$^3$. Determine the allowable gross load on the foundation with a Factor safety (FS) of 4: Assume that the depth of the foundation ($D_f$) is 1.5 m and that general shear failure -occurs in the soil.
Example (2)

Compare *Terzaghi* bearing capacity equations versus a measured field test that resulted in $qu$, if $L = 5.0 \text{ m}$, $c = 0$, $\varphi_t = 42.5^\circ$ and $\gamma' = 9.31 \text{ kN/m}^2$. 

![Diagram of a footing with dimensions and load](image)
General Bearing Capacity Equation
The General Bearing Capacity Equation.

The Terzaghi ultimate bearing capacity equations presented previously are for continuous, square, and circular footings only. They do not include rectangular footings (0 < B/L < 1), or take into account the shearing resistance along the failure surface in the soil above the bottom of the foundation, or the inclination of the footing or the load (Hansen, 1970)

\[ q_u = c'N_c F_{cs} F_{cd} F_{ci} + qN_q F_{qs} F_{qd} F_{qi} + \frac{1}{2} \gamma B N_\gamma F_{\gamma s} F_{\gamma d} F_{\gamma i} \]

Where
- \( c \) = the cohesion;
- \( q \) = the excavated soil’s pressure at the footing’s invert (its bottom);
- \( \gamma \) = the unit weight of the soil;
- \( B \) = width of foundation (equal to the diameter for a circular foundation);
- \( N_c, N_q, N_\gamma \) are the bearing capacity factors;
- \( F_{cs}, F_{qs}, F_{\gamma s} \) are the shape factors;
- \( F_{cd}, F_{qd}, F_{\gamma d} \) are the depth factors; and
- \( F_{ci}, F_{qi}, F_{\gamma i} \) are the load inclination factors.
bearing capacity factors

\( N_q = e^{\pi \tan \phi} \tan^2 \left( 45^\circ + \phi' / 2 \right) \)
\( N_c = \left( N_q - 1 \right) \cot \phi' \)
\( N_y = 2 \left( N_q + 1 \right) \tan \phi' \)

(Table 4.2)

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Shape and Depth, and Inclination Factors

**Shape Factors.**

\[ F_{cs} = 1 + \frac{B}{L} \frac{N_q}{N_c} \]
\[ F_{qs} = 1 + \frac{B}{L} \tan \phi \]
\[ F_{\gamma s} = 1 - 0.4 \frac{B}{L} \]

**Depth Factors for \( D_f / B \leq 1. \)**

\[ F_{cd} = 1 + 0.4 \frac{D_f}{B} \]
\[ F_{qd} = 1 + 2 \tan \phi (1 - \sin \phi)^2 \frac{D_f}{B} \]
\[ F_{\gamma d} = 1 \]

**Depth Factors for \( D_f / B > 1. \)**

\[ F_{cd} = 1 + 0.4 \tan^{-1} \left( \frac{D_f}{B} \right) \]
\[ F_{qd} = 1 + 2 \tan \phi (1 - \sin \phi)^2 \tan^{-1} \frac{D_f}{B} \]
\[ F_{\gamma d} = 1 \]

**Inclination Factors with \( \beta \) is the inclination of load with respect to the vertical.**

\[ F_{ci} = F_{qi} = \left( 1 - \frac{\beta^o}{90^o} \right)^2 \]
\[ F_{\gamma i} = \left( 1 - \frac{\beta}{\phi} \right)^2 \]
Example

A square foundation (B x B) has to be constructed as shown in Figure assume that $\gamma = 17\text{kN/m}^3$, $\gamma_{\text{sat}} = 19.5\text{ kN/m}^3$, $D_1 = 0.75\text{m}$, and $D_f = 1.2\text{m}$. The gross design allowable load, $Q_{\text{all}}$, with FS = 3 is 60 kN. The SPT values are

<table>
<thead>
<tr>
<th>Depth (m)</th>
<th>$N_{60}$ (Blows/ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>4</td>
</tr>
<tr>
<td>1.5</td>
<td>6</td>
</tr>
<tr>
<td>3.0</td>
<td>10</td>
</tr>
<tr>
<td>4.0</td>
<td>5</td>
</tr>
</tbody>
</table>
Foundations with a One-Way Eccentricity.

- In most instances, foundations are subjected to moments in addition to the vertical load as shown below.
- In such cases the distribution of pressure by the foundation upon the soil is not uniform.

The effective width is now, 
\[ B' = B - 2e \]
whereas the effective length is still, 
\[ L' = L \]
The distribution of the nominal (contact) pressure

\[ q_{\text{max}} = \frac{Q}{BL} + \frac{6M}{B^2L} \]

\[ q_{\text{min}} = \frac{Q}{BL} - \frac{6M}{B^2L} \]

where \( Q \) is the total vertical load and \( M \) is the moment on the footing in one axis.

\[ e = \frac{M}{Q} \]

Substituting equation in equations above Eqs. yields:

\[ q_{\text{max}} = \frac{Q}{BL} \left( 1 + \frac{6e}{B} \right) \]

\[ q_{\text{min}} = \frac{Q}{BL} \left( 1 - \frac{6e}{B} \right) \]
• Note that in these equations,
  – when the eccentricity $e$ becomes $B/6$, $q_{\text{min}}$ is zero.
  – For $e > B/6$, $q_{\text{min}}$ will be negative, which means that tension will develop. Because soils can sustain very little tension, there will be a separation between the footing and the soil under it.
  – The value of $q_{\text{max}}$ is then

\[
q_{\text{max}} = \frac{4Q}{3L(B - 2e)}
\]

• Also note that the eccentricity tends to decrease the load bearing capacity of a foundation.
Foundations with Two-way Eccentricities

Consider a footing subject to a vertical ultimate load $Q_{ult}$ and a moment $M$ as shown in Figures. For this case, the components of the moment $M$ about the $x$ and $y$ axis are $M_x$ and $M_y$ respectively. This condition is equivalent to a load $Q$ placed eccentrically on the footing with $x = eB$ and $y = eL$.

$$e_B = \frac{M_y}{Q_{ult}} \quad \text{and} \quad e_L = \frac{M_x}{Q_{ult}}$$
Modification for General Bearing Capacity

The general bearing capacity equation is therefore modified to,

\[ q'_u = c'N_cF_{cs}F_{cd}F_{ci} + qN_qF_{qs}F_{qd}F_{qi} + \frac{1}{2}\gamma B'N_\gamma F_\gamma s F_\gamma d F_\gamma i \]

\[ Q_{ult} = \frac{A'}{q'_u(B')(L')} \]

\[ B' = B - 2e_y \]

\[ L' = L - 2e_x \]

- As before, to evaluate \( F_{cs} \), \( F_{qs} \), and \( F_\gamma s \), use the effective length \((L')\) and the effective width \((B')\) dimensions instead of \(L\) and \(B\), respectively.
- To calculate \( F_{cd} \), \( F_{qd} \), and \( F_\gamma d \) and ,do not replace \( B \) with \( B' \).
- The factor of safety against bearing capacity failure is \( FS = Q_{ult}/Q \)
- Check the factor of safety against \( q_{\text{max}} \) or \( FS = q'_u/q_{\text{max}} \)
- Finally note we confine here our self to \( e_L \leq L/6 \) or \( e_B \leq B/6 \)
Example

A square footing is 1.8 X 1.8 m with a 0.4 X 0.4 m square column. It is loaded with an axial load of 1800 kN and $M_x = 450$ kN • m; $M_y = 360$ kN • m. Undrained triaxial tests (soil not saturated) give $\phi' = 36^\circ$ and $c = 20$ kPa. The footing depth $D = 1.8$ m; the soil unit weight $\gamma = 18.00$ kN/m³; the water table is at a depth of 6.1 m from the ground surface.
Bearing Capacity For Footings On Layered Soils

• There are three general cases of the footing on a layered soil as follows:

Case 1. Footing on layered clays (all $\phi = 0$) as in Fig..
   a. Top layer weaker than lower layer ($c_1 < c_2$)
   b. Top layer stronger than lower layer ($c_1 > c_2$)

Case 2. Footing on layered $\phi$-$c$ soils with $a, b$ same as case 1.

Case 3. Footing on layered sand and clay soils as in Fig.
   a. Sand overlying clay
   b. Clay overlying sand
If \( H \), the thickness of the layer of soil below the footing, is relatively large then the failure surface will be completely located in the top soil layer, which is the upper limit for the ultimate bearing capacity.
If $H$ is small compared to the foundation width $B$, a punching shear failure will occur in the top soil layer followed by a general shear failure in the bottom soil layer.
In this condition, where the stronger surface soil is underlain by a weaker stratum, the *general Bearing capacity* equation is modified to,

\[
q_u = q_b + \left(1 + \frac{B}{L}\right) \left(\frac{2c_a H}{B}\right) + \gamma_1 H^2 \left(1 + \frac{B}{L}\right) \left(1 + \frac{2D_f}{H}\right) \left(\frac{K_s \tan \phi_2}{B}\right) - \gamma_1 H < q_t
\]

where,

- \( c_a \) is the adhesion,
- \( K_s \) is the punching shear coefficient,
- \( q_t \) is the bearing capacity of the top soil layer,
- \( q_b \) is the bearing capacity of the bottom soil layer,
- \( H \) is the height of top layer,
- \( \phi_1 \) is the angle of internal friction of top soil, and
- \( \phi_2 \) for the bottom soil.
Ca determination

\[
\frac{c_a}{c_1} \quad \frac{q_2}{q_1}
\]
punching shear coefficient $K_s$
Example

A foundation 1.5 m by 1 m is placed at a depth of 1 m in a stiff clay. A softer clay layer is located at a depth of 1 m measured from the bottom of the foundation. For the top layer, the un-drained shear strength is 120 kN/m², the unit weight is 16.8 kN/m², and for the bottom layer the un-drained shear strength is 48 kN/m², and the unit weight is 16.2 kN/m². Find the allowable bearing capacity for this footing.
The Other Cases

1. The top layer is strong, and the bottom layer is a saturated soft clay ($\phi = 0$);
2. The top layer is stronger sand and the bottom layer is a weaker sand ($c_1 = 0$) ($c_2 = 0$);
3. The top layer is a stronger saturated clay ($\phi_1 = 0$), and the bottom layer is weaker saturated clay ($\phi_2 = 0$).

Use the same method before and apply corrections were needed
Bearing Capacity From SPT

• Two Ways:
  1. Using the correlation to find $\phi'$ and using the general bearing capacity equation
  2. Using the following chart (for surface footing)
Bearing Capacity From SPT

Allowable bearing capacity for **surface-loaded** footings with settlement limited to approximately 25 mm.
Bearing Capacity From SPT

\[ q_{\text{net(all)}} = 19.16N_{60}F_d \left( \frac{S_a}{25.4} \right) \]  
\[ \text{For } B \leq 1.22 \text{ m} \]

\[ q_{\text{net(all)}} = 11.98N_{60}(3.28B + 1)^2 F_d \left( \frac{S_a}{25.4} \right) \]  
\[ \text{For } B \geq 1.22 \text{ m} \]

Where

\[ q_{\text{net(all)}} = q_{\text{all}} - \gamma D_f \text{ kn/m}^2 \]

Sa: tolerable settlement in mm

Fd=depth factor=1+0.33(Df/B)≤1.33

Bearing Capacity Using The Cone Penetration Test (CPT)

\[ q_{\text{net(all)}} = \left( \frac{q_c}{15} \right) \]  
\[ \text{For } B \leq 1.22 \text{ m} \]

\[ q_{\text{net(all)}} = \left( \frac{q_c}{25} \right)(3.28B + 1)^2 \]  
\[ \text{For } B \geq 1.22 \text{ m} \]

Where

\[ q_{\text{net(all)}} = q_{\text{all}} - \gamma D_f \text{ kn/m}^2 \]
Determine the size of the square footing, if the soil has a $\gamma = 105$ pcf, $\gamma_{sat} = 118$ pcf, $D_f = 4$ feet, $D_I = 2$ ft. The gross design load $Q_{all}$ is 150 kips with a FS = 3. The field SPTs are as follows,

<table>
<thead>
<tr>
<th>Depth(ft)</th>
<th>N (blows/ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>10</td>
<td>6</td>
</tr>
<tr>
<td>15</td>
<td>6</td>
</tr>
<tr>
<td>20</td>
<td>10</td>
</tr>
<tr>
<td>25</td>
<td>5</td>
</tr>
</tbody>
</table>
The Bearing Capacity of Mat Foundations
Mat foundations must be designed to limit their settlements to a tolerable amount.

The ultimate bearing capacity of a soil supporting a mat foundation can be computed from,

\[ q_u = c N_c \, F_{cs} \, F_{cd} \, F_{ci} + \gamma D_f N_q \, F_{qs} \, F_{qd} \, F_{qi} + \gamma B N_\gamma \, F_{\gamma s} \, F_{\gamma d} \, F_{\gamma i} \]

When \( \phi = 0 \) use,

\[ q_u = 5.14 c_u \left(1 + 0.195 \frac{B}{L}\right) \left(1 + 0.4 \frac{D_f}{B}\right) \]

where \( c_u \) is the un-drained cohesion. When using corrected SPT values, the allowable bearing capacity may be calculated by,

\[ q_{all} = 11.98 N \left(1 + 0.33 \frac{D_f}{B}\right) \left(\frac{S}{25.4}\right) < 15.93 \left(\frac{S}{25.4}\right) \]

where \( N \) is the corrected standard penetration resistance, and \( s \) is the settlement in millimeters.

\[ q_{all} \text{ (in } kN/m^2) = 36N (1+0.33D_f)\left(\frac{\Delta}{25.4}\right) \]
The depth of embedment $D_f$ for fully compensated foundation is,

$$D_f = \frac{Q_u}{\gamma A}$$
Bearing Capacity for Field Load Tests

Dead weight of a truck or a beam attached to anchor piles

Props for stability when using dead weights.

Jack

Short block

Steel plate

anchors or piles to provide a reactive force

Several dial gauges attached to an independent suspension system to record plate settlements with each increment of the jack load.
Bearing Capacity Based On Building Codes
(Presumptive Pressure)

<table>
<thead>
<tr>
<th>Soil description</th>
<th>Presumptive bearing capacities from indicated building codes, kPa</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clay, very soft</td>
<td>25</td>
</tr>
<tr>
<td>Clay, soft</td>
<td>75</td>
</tr>
<tr>
<td>Clay, ordinary</td>
<td>125</td>
</tr>
<tr>
<td>Clay, medium stiff</td>
<td>175</td>
</tr>
<tr>
<td>Clay, stiff</td>
<td>210</td>
</tr>
<tr>
<td>Clay, hard</td>
<td>300</td>
</tr>
<tr>
<td>Sand, compact and clean</td>
<td>240</td>
</tr>
<tr>
<td>Sand, compact and silty</td>
<td>100</td>
</tr>
<tr>
<td>Inorganic silt, compact</td>
<td>125</td>
</tr>
<tr>
<td>Sand, loose and fine</td>
<td>140</td>
</tr>
<tr>
<td>Sand, loose and coarse, or sand-gravel mixture, or compact and fine</td>
<td>140 to 400</td>
</tr>
<tr>
<td>Gravel, loose and compact coarse sand</td>
<td>300</td>
</tr>
<tr>
<td>Sand-gravel, compact</td>
<td>240</td>
</tr>
<tr>
<td>Hardpan, cemented sand, cemented gravel</td>
<td>600</td>
</tr>
<tr>
<td>Soft rock</td>
<td></td>
</tr>
<tr>
<td>Sedimentary layered rock</td>
<td>6000</td>
</tr>
<tr>
<td>(hard shale, sandstone, siltstone)</td>
<td></td>
</tr>
<tr>
<td>Bedrock</td>
<td>9600</td>
</tr>
</tbody>
</table>

Note: Values converted from psf to kPa and rounded.

*Building Officials and Code Administrators International, Inc.
Safety Factors In Foundation Design

There are more uncertainties in determining the allowable strength of the soil than in the superstructure elements. These may be summarized as follows:

• Complexity of soil behavior
• Lack of control over environmental changes after construction
• Incomplete knowledge of subsurface conditions
• Inability to develop a good mathematical model for the foundation
• Inability to determine the soil parameters accurately
Safety Factors In Foundation Design

These uncertainties and resulting approximations have to be evaluated for each site and a suitable safety factor directly (or indirectly) assigned that is not overly conservative but that takes into account at least the following:

1. Magnitude of damages (loss of life, property damage, and lawsuits) if a failure results
2. Relative cost of increasing or decreasing SF
3. Relative change in probability of failure by changing SF
4. Reliability of soil data
5. Changes in soil properties from construction operations, and later from any other causes
6. Accuracy of currently used design/analysis methods
Safety Factors Usually Used

- **Values of stability numbers** (or safety factors) usually used
- It is customary to use overall safety factors on the order of those shown in Table. Shear should be interpreted as bearing capacity for footings.

<table>
<thead>
<tr>
<th>Failure mode</th>
<th>Foundation type</th>
<th>SF</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shear</td>
<td>Earthworks Dams, fills, etc.</td>
<td>1.2–1.6</td>
</tr>
<tr>
<td>Shear</td>
<td>Retaining structure Walls</td>
<td>1.5–2.0</td>
</tr>
<tr>
<td>Shear</td>
<td>Sheetpiling cofferdams Temporary braced excavations</td>
<td>1.2–1.6</td>
</tr>
<tr>
<td>Shear</td>
<td>Footings Spread Mat Uplift Uplift, heaving Piping</td>
<td>2–3 1.7–2.5 1.7–2.5 1.5–2.5 3–5</td>
</tr>
</tbody>
</table>
## Bearing Capacity Of Rock

<table>
<thead>
<tr>
<th>Type of rock</th>
<th>Typical unit wt., kN/m³</th>
<th>Modulus of elasticity $E$, MPa $\times 10^3$</th>
<th>Poisson’s ratio, $\mu$</th>
<th>Compressive strength, MPa</th>
</tr>
</thead>
<tbody>
<tr>
<td>Basalt</td>
<td>28</td>
<td>17–103</td>
<td>0.27–0.32</td>
<td>170–415</td>
</tr>
<tr>
<td>Granite</td>
<td>26.4</td>
<td>14–83</td>
<td>0.26–0.30</td>
<td>70–276</td>
</tr>
<tr>
<td>Schist</td>
<td>26</td>
<td>7–83</td>
<td>0.18–0.22</td>
<td>35–105</td>
</tr>
<tr>
<td>Limestone</td>
<td>26</td>
<td>21–103</td>
<td>0.24–0.45</td>
<td>35–170</td>
</tr>
<tr>
<td>Porous limestone</td>
<td></td>
<td>3–83</td>
<td>0.35–0.45</td>
<td>7–35</td>
</tr>
<tr>
<td>Sandstone</td>
<td>22.8–23.6</td>
<td>3–42</td>
<td>0.20–0.45</td>
<td>28–138</td>
</tr>
<tr>
<td>Shale</td>
<td>15.7–22</td>
<td>3–21</td>
<td>0.25–0.45</td>
<td>7–40</td>
</tr>
<tr>
<td>Concrete</td>
<td>15.7–23.6</td>
<td>Variable</td>
<td>0.15</td>
<td>15–40</td>
</tr>
</tbody>
</table>

*Depends heavily on confining pressure and how determined; $E =$ tangent modulus at approximately 50 percent of ultimate compression strength.

The bearing-capacity factors for sound rock are approximately

$$N_q = \tan^6 \left( 45^\circ + \frac{\phi}{2} \right) \quad N_c = 5 \tan^4 \left( 45^\circ + \frac{\phi}{2} \right) \quad N_\gamma = N_q + 1$$

$$q'_{ult} = q_{ult} (RQD)^2$$
References