Chapter 10 Flow in Conduits

10.1 Classifying Flow
Laminar Flow and Turbulent Flow

<table>
<thead>
<tr>
<th>Re</th>
<th>Laminary Flow</th>
<th>Unpredictable</th>
<th>Turbulent Flow</th>
</tr>
</thead>
<tbody>
<tr>
<td>≤ 2000</td>
<td>Laminar flow</td>
<td>2000 ≤ Re ≤ 3000</td>
<td>Unpredictable</td>
</tr>
<tr>
<td>≥ 3000</td>
<td>Turbulent flow</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ Re = \frac{\rho UD}{\mu} = \frac{UD}{v}, \quad v = \frac{\mu}{\rho} \]

Near entrance: undeveloped “developing” flow
In developing flow, the wall shear stress is changing. In fully developed flow, the wall shear stress is constant.

10.3 Pipe Head Loss
Combined (Total) Head Loss

Total Head Loss = pipe head loss + component head loss
Component head loss is associated with flow through devices such as valves, bends, and tees. Pipe head loss is associated with fully developed flow in conduits, and it is caused by shear stresses that act on the flowing fluid.

The Darcy-Weisbach equation, the flow should be fully developed and steady:

\[ f = \frac{4\tau_0}{\rho V^2/2} \approx \text{shear stress acting on wall} \]

\[ h_f = f \frac{L V^2}{D 2g} \]

10.5 Laminar Flow in a Round Tube
For laminar flow

\[ V(r) = V_{max}(1 - \frac{r}{r_0})^2 \]

\[ V_{max} = -\left( \frac{r_0}{4\mu} \right) \left( \frac{\gamma \Delta h}{\Delta L} \right) \]

\[ r_0: \text{radius of pipe, } \Delta h \text{ is the change in piezometric head over a length } \Delta L \text{ of conduit.} \]

\[ \bar{V} = -\left( \frac{D^2}{32\mu} \right) \left( \frac{\gamma \Delta h}{\Delta L} \right) = \frac{V_{max}}{2} \]

\[ h_f = f \frac{L V^2}{D 2g} \text{ where } f = \frac{64}{Re} \]

10.6 Turbulent Flow and the Moody Diagram
Turbulent flow is a flow regime in which the movement of fluid particles is chaotic, eddying, and unsteady, with significant movement of particles in directions transverse to the flow direction.

Because of the chaotic motion of fluid particles, turbulent flow produces high levels of mixing and has a velocity profile that is more uniform or flatter than the corresponding laminar velocity profile. Engineers and scientists model turbulent flow by using an empirical approach. Because the complex nature of turbulent flow has prevented researchers from establishing a mathematical solution of general utility.
Equations for the Velocity Distribution

\[ \frac{V(r)}{V_{max}} = \left( \frac{r_0 - r}{r_0} \right)^m \]

<table>
<thead>
<tr>
<th>( \text{Re} )</th>
<th>( 4 \times 10^3 )</th>
<th>( 2.3 \times 10^4 )</th>
<th>( 1.1 \times 10^5 )</th>
<th>( 1.1 \times 10^6 )</th>
<th>( 3.2 \times 10^6 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m )</td>
<td>( \frac{1}{6} )</td>
<td>( \frac{1}{6.6} )</td>
<td>( \frac{1}{7} )</td>
<td>( \frac{1}{8.8} )</td>
<td>( \frac{1}{10} )</td>
</tr>
<tr>
<td>( \frac{u_{max}}{V} )</td>
<td>( 1.26 )</td>
<td>( 1.24 )</td>
<td>( 1.22 )</td>
<td>( 1.18 )</td>
<td>( 1.16 )</td>
</tr>
</tbody>
</table>

the turbulent boundary-layer equations

\[ \frac{V(r)}{V^*} = 2.44 \ln \frac{(V^*(r_0 - r))}{v} + 5.56 \]

\( V^* \): shear velocity = \( \sqrt{\tau_0/\rho} \)

Equations for the Friction Factor, \( f \)

the resistance coefficient for turbulent flow in tubes that have smooth walls

\[ \frac{1}{\sqrt{f}} = 2.0 \log_{10}(Re \sqrt{f}) - 0.8 \]

<table>
<thead>
<tr>
<th>Type of Flow</th>
<th>Parameter Ranges</th>
<th>Influence of Parameters on ( f )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Laminar Flow</td>
<td>( \text{Re} \leq 2000 )</td>
<td>( f ) is independent of Reynolds number</td>
</tr>
<tr>
<td>Turbulent Flow, Smooth Tube</td>
<td>( \text{Re} &gt; 3000 )</td>
<td>( f ) depends on Reynolds number, ( (k/D) )</td>
</tr>
<tr>
<td>Transitional Turbulent Flow</td>
<td>( \text{Re} &gt; 3000 )</td>
<td>( f ) depends on Reynolds number, ( (k/D) )</td>
</tr>
<tr>
<td>Fully Rough Turbulent Flow</td>
<td>( \text{Re} &gt; 3000 )</td>
<td>( f ) depends on Reynolds number, ( (k/D) )</td>
</tr>
</tbody>
</table>

Moody Diagram

<table>
<thead>
<tr>
<th>Boundary Material</th>
<th>( k_p ), Millimeters</th>
<th>( k_p ), Inches</th>
</tr>
</thead>
<tbody>
<tr>
<td>Glass, plastic</td>
<td>Smooth</td>
<td>Smooth</td>
</tr>
<tr>
<td>Copper or brass tubing</td>
<td>0.0015</td>
<td>( 6 \times 10^{-5} )</td>
</tr>
<tr>
<td>wrought iron, steel</td>
<td>0.046</td>
<td>0.002</td>
</tr>
<tr>
<td>asphalted cast iron</td>
<td>0.12</td>
<td>0.005</td>
</tr>
<tr>
<td>Galvanized iron</td>
<td>0.15</td>
<td>0.006</td>
</tr>
<tr>
<td>cast iron</td>
<td>0.26</td>
<td>0.010</td>
</tr>
<tr>
<td>Concrete</td>
<td>0.3 to 3.0</td>
<td>0.012–0.12</td>
</tr>
<tr>
<td>Riveted steel</td>
<td>0.9–9</td>
<td>0.035–0.35</td>
</tr>
<tr>
<td>rubber pipe (straight)</td>
<td>0.025</td>
<td>0.001</td>
</tr>
</tbody>
</table>
To provide a more convenient solution to some types of problems, the top of the Moody diagram presents a scale based on the parameter $Re f^{1/2}$. This parameter is useful when $hf$ and $ks/D$ are known but the velocity $V$ is not.

In the Moody diagram, the variable $ks$ denotes the *equivalent sand roughness*. That is, a pipe that has the same resistance characteristics at high $Re$ values as a sand-roughened pipe is said to have a roughness equivalent to that of the sand-roughened pipe. Table 10.4 gives the equivalent sand roughness for various kinds of pipes. This table can be used to calculate the relative roughness for a given pipe diameter, which, in turn, is used in Fig."Moody chart" to find the friction factor.

Using the Darcy-Weisbach equation and the definition of Reynolds number

$$Re f^{1/2} = \frac{D^3}{v} \left(\frac{2gh_f}{L}\right)^{1/2}$$

In the Moody diagram, curves of constant $Re f^{1/2}$ are plotted using heavy black lines that slant from the left to right.

$$h_f = f \frac{L V^2}{D 2g}$$

When using computers to carry out pipe-flow calculations, it is much more convenient to have an equation for the friction factor as a function of Reynolds number and relative roughness.
\[
f = \frac{0.25}{\log_{10} \left( \frac{k_s}{3.7D} + \frac{5.74}{Re^{0.5}} \right)}
\]

It is reported that this equation predicts friction factors that differ by less than 3% from those on the Moody diagram for \(4 \times 10^3 < Re < 10^8\) and \(10^{-3} < ks/D < 2 \times 10^{-2}\).

### 10.8 Combined Head Loss

**The Minor Loss Coefficient, \(K\)**

When fluid flows through a component such as a partially open valve or a bend in a pipe, viscous effects cause the flowing fluid to lose mechanical energy.

\[
K = \frac{\Delta h}{V^2/2g} = \frac{\Delta p}{(\rho V^2)/2}
\]

\(\Delta h\): drop in piezometric head that is caused by a component.

\(\Delta p\): drop in pressure that is caused by a component.

\(V\): Mean velocity

\[
h_L = \frac{K V^2}{2g}
\]

<table>
<thead>
<tr>
<th>Description</th>
<th>Sketch</th>
<th>Additional Data</th>
<th>(K)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Pipe entrance</strong></td>
<td></td>
<td>(r/d)</td>
<td>(K_0)</td>
</tr>
<tr>
<td>(h_L = K_0 V^2/2g)</td>
<td></td>
<td>0.0</td>
<td>0.50</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.1</td>
<td>0.12</td>
</tr>
<tr>
<td></td>
<td></td>
<td>&gt;0.2</td>
<td>0.03</td>
</tr>
<tr>
<td><strong>Contraction</strong></td>
<td></td>
<td>(D_2/D_1)</td>
<td>(K_C)</td>
</tr>
<tr>
<td>(h_L = K_C V^2/2g)</td>
<td></td>
<td>0.00</td>
<td>0.08</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.20</td>
<td>0.08</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.40</td>
<td>0.07</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.60</td>
<td>0.06</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.80</td>
<td>0.06</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.90</td>
<td>0.06</td>
</tr>
<tr>
<td><strong>Expansion</strong></td>
<td></td>
<td>(D_1/D_2)</td>
<td>(K_E)</td>
</tr>
<tr>
<td>(h_L = K_E V^2/2g)</td>
<td></td>
<td>0.00</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.20</td>
<td>0.30</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.40</td>
<td>0.25</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.60</td>
<td>0.15</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.80</td>
<td>0.10</td>
</tr>
<tr>
<td><strong>90° miter bend</strong></td>
<td></td>
<td></td>
<td>(K_0 = 1.1)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>With vanes</td>
<td>(K_0 = 0.2)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(r/d)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>1</td>
<td>0.35</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>0.19</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4</td>
<td>0.16</td>
</tr>
<tr>
<td></td>
<td></td>
<td>6</td>
<td>0.21</td>
</tr>
</tbody>
</table>
Threaded pipe fittings
- Globe valve—wide open \( K_v = 10.0 \)
- Angle valve—wide open \( K_v = 5.0 \)
- Gate valve—wide open \( K_v = 0.2 \)
- Gate valve—half open \( K_v = 5.6 \)
- Return bend \( K_b = 2.2 \)
- Tee
- Straight-through flow \( K_t = 0.4 \)
- Side-outlet flow \( K_t = 1.8 \)
- 90° elbow \( K_\varnothing = 0.9 \)
- 45° elbow \( K_\varnothing = 0.4 \)

Combined Head Loss Equation
Total head loss = \{Pipe head loss\} + \{Component head loss\}

\[ h_L = \sum_{\text{Pipes}} f \frac{L V^2}{D 2g} + \sum_{\text{components}} K \frac{V^2}{2g} \]

10.9 Non-round Conduits

\[ h_L = f \frac{L V^2}{D_h 2g} \]

\[ D_h = \frac{4 \times \text{cross section area}}{\text{wetted perimeter}} \]

For rectangular cross section: \( L \times w \), area = \( L \times w \), perimeter = \( 2L + 2w = 2(L + w) \)

\[ D_h = \frac{4(L \times w)}{2(L + w)} = \frac{2Lw}{L + w} \]
The head loss per kilometer of 20 cm asphalted cast-iron pipe is 12.2 m. What is the flow rate of water through the pipe?

1. Compute the parameter $D^{3/2} \sqrt{2gh_f / L / v}$.

\[
D^{3/2} \sqrt{2gh_f / L / v} = (0.20 \text{ m})^{3/2} \times \frac{[2(9.81 \text{ m/s}^2)(12.2 \text{ m} / 1000 \text{ m})]}{1.0 \times 10^{-6} \text{ m}^2 / \text{s}} = 4.38 \times 10^4
\]

2. Determine resistance coefficient.

- Relative roughness:
  \[ k_s / D = (0.00012 \text{ m}) / (0.2 \text{ m}) = 0.0006 \]
- Look up $f$ on the Moody diagram for $D^{3/2} \sqrt{2gh_f / L / v} = 4.4 \times 10^4$ and $k_s / D = 0.0006$: $f = 0.019$

3. Find $V$ using the Darcy-Weisbach equation.

\[
h_f = f \left( \frac{L}{D} \right) \left( \frac{V^2}{2g} \right)
\]

\[
12.2 \text{ m} = 0.019 \left( \frac{1000 \text{ m}}{0.2 \text{ m}} \right) \left( \frac{V^2}{2(9.81 \text{ m/s}^2)} \right)
\]

\[ V = 1.59 \text{ m/s} \]

4. Use flow rate equation to find discharge.

\[
Q = VA = (1.59 \text{ m/s}) \left( \frac{\pi}{4} \right)(0.2 \text{ m})^2 = 0.05 \text{ m}^3 \text{/s}
\]
Find: Power required to operate pump.

Properties: From Table A.5 \( \nu = 6.58 \times 10^{-7} \text{ m}^2/\text{s} \).
From Table 10.2 \( k_s = 0.0015 \text{ mm} \).

**ANALYSIS**

**Reynolds number**

\[
\text{Re} = \frac{0.02 \times 10}{6.58 \times 10^{-7}} = 3.04 \times 10^5
\]

**Flow rate equation**

\[
Q = \frac{\pi}{4} \times 0.02^2 \times 10 = 0.00314 \text{ m}^3/\text{s}
\]

**Relative roughness (copper tubing)**

\[
\frac{k_s}{D} = \frac{1.5 \times 10^{-3} \text{ mm}}{20 \text{ mm}} = 7.5 \times 10^{-5}
\]

**Resistance coefficient (from Moody diagram)**

\[
f = 0.0155
\]

**Energy equation**

\[
h_p = \frac{V^2}{2g} (f \frac{L}{D} + \sum K_f)
= \frac{10^2}{2 \times 9.81} (0.0155 \times \frac{10 \text{ m}}{0.02 \text{ m}} + 14 \times 2.2) = 196 \text{ m}
\]

**Power equation**

\[
P = \frac{\gamma Qh_p}{\eta}
= \frac{9732 \times 0.00314 \times 196}{0.8}
= 7487 \text{ W}
\]

\[P = 7.49 \text{ kW}\]
Q10.66 What power must the pump supply to the system to pump the oil from the lower reservoir to the upper reservoir at a rate of 0.20 m³/s? Sketch the HGL and the EGL for the system.

From Table 10.2 \( k_s = 0.048 \text{ mm} \)

Energy equation

\[
p_1/\gamma + \alpha_1 V_1^2/2g + z_1 + h_p = p_2/\gamma + \alpha_2 V_2^2/2g + z_2 + \sum h_L
\]

\[
100 + h_p = 112 + V^2/2g(K_s + fL/D + K_E)
\]

\[
h_p = 12 + (V^2/2g)(0.03 + fL/D + 1)
\]

Flow rate equation

\[
V = Q/A
\]

\[
= 0.20/((\pi/4) \times 0.30^2)
\]

\[
= 2.83 \text{ m/s}
\]

\[
V^2/2g = 0.408 \text{ m}
\]

Reynolds number

\[
Re = VD/\nu
\]

\[
= 2.83 \times 0.30/(10^{-5})
\]

\[
= 8.5 \times 10^4
\]

\[
k_s/D = 4.6 \times 10^{-5}/0.3
\]

\[
= 1.5 \times 10^{-4}
\]

Resistance coefficient (from the Moody diagram, Fig. 10.8)

\[
f = 0.019
\]

Then

\[
h_p = 12 + 0.408(0.03 + (0.019 \times 150/0.3) + 1.0)
\]

\[
= 16.3 \text{ m}
\]

Power equation

\[
P = Q\gamma h_p
\]

\[
= 0.20 \times (940 \times 9.81) \times 16.3 = 2.67 \times 10^4 \text{ W}
\]

\[
= 30.1 \text{ kW}
\]